Mr. Wright's Math Extravaganza

Precalculus

Sequences, Series, and Probability

Level 2.0: 70% on test, Level 3.0: 80% on test, Level 4.0: level 3.0 and success on applications

Score	I Can Statements
4.0	☐ I can demonstrate in-depth inferences and applications that go beyond what was taught.
3.5	In addition to score 3.0 performance, partial success at score 4.0 content
3.0	☐ I can write a rule for a sequence. ☐ I can evaluate a summation. ☐ I can apply the fundamental counting principle. ☐ I can calculate probabilities.
2.5	No major errors or omissions regarding score 2.0 content, and partial success at score 3.0 content
2.0	☐ I can write a sequence from a rule. ☐ I can write a series as a summation. ☐ I can write and evaluate arithmetic sequences and series. ☐ I can write and evaluate geometric sequences and series. ☐ I can use mathematical induction to prove sum formulas and other mathematical statements. ☐ I can expand binomial expressions using the binomial theorem. ☐ I can calculate permutations and combinations. ☐ I can calculate probabilities of simple events, compound events, and multiple events.
1.5	Partial success at score 2.0 content, and major errors or omissions regarding score 3.0 content.
1.0	With help, partial success at score 2.0 content and score 3.0 content.
0.5	With help, partial success at score 2.0 content but not at score 3.0 content.
0.0	Even with help, no success.

Richard Wright

Precalculus

10-01 Sequences

Sequence

- List of numbers following a rule
- 0, 3, 6, 9, 12 ← _____ (ends)
- 0, 3, 6, 9, 12, … ← _____ (doesn't end)

- $n = 1,\ 2,\ 3,\ 4,\ 5,\ \ldots$ (term _____) like *x*
- $a_n = 0,\ 3,\ 6,\ 9,\ 12,\ \ldots$ (term _____) like *y*

Find the 1st 5 terms of $a_n = 5 + 2n(-1)^n$

Write the rule for the n^{th} term.

1, 5, 9, 13, 17, …

2, –9, 28, –65, 126, …

Recursive Rules

- Use the value of one term to find the _____ term.
- a_n means _____ term
- a_{n-1} means _____ term

Find the first 5 terms. $a_1 = 6, a_n = a_{n-1} + 1$

Factorial (!)

- Product of a _____ number with all the _____ numbers _____ than it through 1.
- $6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$
- $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$
- $0!$ = _____

Simplify $\frac{9!}{3!7!}$

$\frac{(n+1)!}{n!}$

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10-02 Series

Series

- _____ of a sequence
- Sequence
  - _____
- Series
  - _____

Summation Notation (Sigma Notation)

$$\sum_{i=1}^{n} a_i = a_1 + a_2 + a_3 + \cdots + a_n$$

Find each sum

$\sum_{i=1}^{4}(4i+1)$

$\sum_{k=2}^{5}(2+k^3)$

$\sum_{n=1}^{\infty}\frac{5}{10^n}$

Shortcut formulas

$$1 + 1 + 1 + 1 + \cdots = \sum_{i=1}^{n} 1 = n$$

$$1 + 2 + 3 + 4 + \cdots = \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$1 + 4 + 9 + 16 + \cdots = \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$1 + 8 + 27 + 64 + \cdots = \sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$$

$$1 + 16 + 81 + 256 + \cdots = \sum_{i=1}^{n} i^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$$

$$1 + 32 + 243 + 1024 + \cdots = \sum_{i=1}^{n} i^5 = \frac{n^2(n+1)^2(2n^2+2n-1)}{12}$$

Evaluate

$$\sum_{i=1}^{5} (3i^2 - 5i)$$

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10-03 Arithmetic Sequences and Series

Arithmetic Sequence

- Common _____ (d)
- 3, 7, 11, 15, 19, …

Rule for the n^{th} term

$$a_n = dn + c$$

Where $c = a_1 - d$

$$a_n = a_1 + (n-1)d$$

Find the rule for the n^{th} term for 3, 7, 11, 15, 19, …

The 8^{th} term of an arithmetic sequence is 25, and the 12^{th} term is 41. Write the rule for the n^{th} term.

Recursive Rule for Arithmetic Sequences

$$a_1 = a_1$$
$$a_n = a_{n-1} + d$$

Arithmetic Series

$$S_n = \frac{n}{2}(a_1 + a_n)$$

Find the sum of the integers 1 to 57.

Find the 50th partial sum of the arithmetic sequence –6, –2, 2, 6, …

Evaluate

$$\sum_{i=1}^{100}(3i + 2)$$

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10-04 Geometric Sequences and Series

Geometric Sequence

- Common _____(r)
- 1, 3, 9, 27, 81, 243, …

Rule for n^{th} term

$$a_n = a_1 r^{n-1}$$

Find the rule for $6, \ -2, \frac{2}{3}, \ \ldots$

The 2nd term of a geometric sequence is −18, the 5th term is 2/3. Find the rule for the n^{th} term.

Geometric Series

$$S_n = a_1\left(\frac{1-r^n}{1-r}\right)$$

$$S_\infty = \frac{a_1}{1-r}$$

where $|r| < 1$

Evaluate

$$\sum_{n=1}^{7} 2^{n-1}$$

Evaluate

$5 + 0.5 + 0.05 + 0.005 + \cdots$

$$\sum_{n=0}^{\infty} 5\left(\frac{1}{2}\right)^n$$

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10-05 Mathematical Induction

- Proofs for _____formulas
- Show it works when _____
- Show it works for _____

Steps for Proof by Induction

1. Show it works for _____
2. _____formula works for $n = k$
3. Show it works for _____

- If proving _____formula use $S_{k+1} = S_k + a_{k+1}$

Prove $5 + 7 + 9 + 11 + 13 + \cdots + (3 + 2n) = n(n + 4)$

Prove $1(1 - 1) + 2(2 - 1) + 3(3 - 1) + \cdots + n(n - 1) = \frac{n(n-1)(n+1)}{3}$

Prove $(n+1)! > 2^n$ where $n \geq 2$

Prove 4 is a factor of $5^n - 1$

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10-06 Binomial Theorem

$(x+y)^0$ 1

$(x+y)^1$ $1x$ $1y$

$(x+y)^2$ $1x^2$ $2xy$ $1y^2$

$(x+y)^3$ $1x^3$ $3x^2y$ $3xy^2$ $1y^3$

$(x+y)^4$ $1x^4$ $4x^3y$ $6x^2y^2$ $4xy^3$ $1y^4$

Properties

1. _____terms
2. Powers of *x* count _____, *y*'s count _____
3. Sum of exponents of each term = _____
4. Coefficients are _____

Binomial theorem

$$(a+b)^n = \sum_{r=0}^{n} {}_nC_r a^{n-r} b^r$$

- where ${}_nC_r = \frac{n!}{(n-r)!r!}$

Evaluate

${}_9C_2$ $\binom{11}{4}$

$\binom{8}{8}$ $\binom{4}{2}$

Expand $(x+2)^4$

Expand $(3 - x^2)^5$

Find the coefficient of the term a^4b^7 in $(2a - 3b)^{11}$

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10-07 Counting Principles

Fundamental Counting Principle

- If events E_1 and E_2 occur in m_1 and m_2 ways, the number of ways _____events can occur is _____.

A lock will open with the right choice of 3 numbers. How many different sets of 3 numbers can you choose if each number is from 1 to 30 inclusive? (a) with repetition (b) without repetition

How many license plates can be made if each is 2 letters follow by 4-digits? (a) with repetition (b) without repetition

Permutation

- Number of ways to _____n objects taken r at a time

$$ {}_nP_r = \frac{n!}{(n-r)!} $$

How many ways can 8 children line up in a row?

A club has 24 members, how many ways can 5 officers be selected?

Distinguishable Permutations

- We want the orders that look _____(choosing _____the objects)

$$ \frac{n!}{q_1! \cdot q_2! \cdot q_3! \cdots} $$

- Where n = number of objects; q = how many times each is repeated

How many distinguishable ways to order the letters in BANANA?

Combinations

- Grouping of objects _____order

$$_nC_r = \frac{n!}{(n-r)!\,r!}$$

There are 31 students. How many different groups of 4 can be made?

You are forming a 10-person committee from 9 women and 12 men. How many different committees if 5 women and 5 men?

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10-08 Probability

Probability

- Number from _____to _____indicating how _____something is to happen.
- 0 = _____happens
- 1 = _____happens

$$P(A) = \frac{favorable\ outcomes}{total\ outcomes}$$

A box contains 3 red marbles, 5 black marbles, and 2 yellow marbles. If a marble is selected at random, what is the probability of choosing yellow?

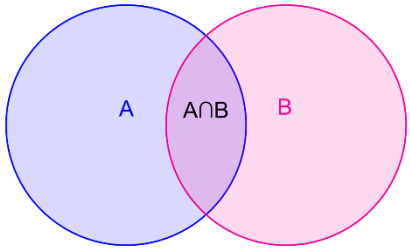
2 dice are rolled, what is the probability that the sum is 8?

Compound Events

- _____event with _____accepted outcomes

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

- If $P(A \cap B) = 0$, then called _____exclusive



You draw one card from a standard 52-card deck. What is the probability of drawing a heart or red?

Multiple Events

- _____events with _____outcomes
- Independent – Event A _____affect event B

$$P(A\ and\ B) = P(A) \cdot P(B)$$

- Dependent – Event A _____affect event B

$$P(A\ and\ B) = P(A) \cdot P(B|A)$$

  - where $P(B|A)$ is the probability that B occurs given that A already occured

You draw 2 cards from a standard 52-card deck. What is the probability you draw a heart and a red? (a) with replacement (b) without replacement

Complement

- _____

$$P(\overline{A}) = 1 - P(A)$$

- _____P(n ≥ 1) is easier with the complement _____$P(0)$