Precalculus

10-01 Sequences

Sequence

- List of numbers following a rule
- 0, 3, 6, 9, 12 ← _____ (ends)
- 0, 3, 6, 9, 12, … ← _____ (doesn't end)

- $n = 1,\ 2,\ 3,\ 4,\ 5,\ \ldots$ (term _____) like *x*
- $a_n = 0,\ 3,\ 6,\ 9,\ 12,\ \ldots$ (term _____) like *y*

Find the 1st 5 terms of $a_n = 5 + 2n(-1)^n$

Write the rule for the n^{th} term.

1, 5, 9, 13, 17, …

2, –9, 28, –65, 126, …

Recursive Rules

- Use the value of one term to find the _____ term.
- a_n means _____ term
- a_{n-1} means _____ term

Find the first 5 terms. $a_1 = 6, a_n = a_{n-1} + 1$

Factorial (!)

- Product of a _____ number with all the _____ numbers _____ than it through 1.
- $6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$
- $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$
- $0!$ = _____

Simplify $\frac{9!}{3!7!}$

$\frac{(n+1)!}{n!}$

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10-02 Series

Series

- _____ of a sequence
- Sequence
  - _____
- Series
  - _____

Summation Notation (Sigma Notation)

$$\sum_{i=1}^{n} a_i = a_1 + a_2 + a_3 + \cdots + a_n$$

Find each sum

$\sum_{i=1}^{4}(4i+1)$

$\sum_{k=2}^{5}(2+k^3)$

$\sum_{n=1}^{\infty}\frac{5}{10^n}$

Shortcut formulas

$$1 + 1 + 1 + 1 + \cdots = \sum_{i=1}^{n} 1 = n$$

$$1 + 2 + 3 + 4 + \cdots = \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$1 + 4 + 9 + 16 + \cdots = \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$1 + 8 + 27 + 64 + \cdots = \sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$$

$$1 + 16 + 81 + 256 + \cdots = \sum_{i=1}^{n} i^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$$

$$1 + 32 + 243 + 1024 + \cdots = \sum_{i=1}^{n} i^5 = \frac{n^2(n+1)^2(2n^2+2n-1)}{12}$$

Evaluate

$$\sum_{i=1}^{5} (3i^2 - 5i)$$

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10-03 Arithmetic Sequences and Series

Arithmetic Sequence

- Common _____ (d)
- 3, 7, 11, 15, 19, …

Rule for the n^{th} term

$$a_n = dn + c$$

Where $c = a_1 - d$

$$a_n = a_1 + (n-1)d$$

Find the rule for the n^{th} term for 3, 7, 11, 15, 19, …

The 8^{th} term of an arithmetic sequence is 25, and the 12^{th} term is 41. Write the rule for the n^{th} term.

Recursive Rule for Arithmetic Sequences

$$a_1 = a_1$$
$$a_n = a_{n-1} + d$$

Arithmetic Series

$$S_n = \frac{n}{2}(a_1 + a_n)$$

Find the sum of the integers 1 to 57.

Find the 50th partial sum of the arithmetic sequence –6, –2, 2, 6, …

Evaluate

$$\sum_{i=1}^{100}(3i + 2)$$

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10-04 Geometric Sequences and Series

Geometric Sequence

- Common _____(r)
- 1, 3, 9, 27, 81, 243, …

Rule for n^{th} term

$$a_n = a_1 r^{n-1}$$

Find the rule for $6, \ -2, \frac{2}{3}, \ \ldots$

The 2nd term of a geometric sequence is −18, the 5th term is 2/3. Find the rule for the n^{th} term.

Geometric Series

$$S_n = a_1\left(\frac{1-r^n}{1-r}\right)$$

$$S_\infty = \frac{a_1}{1-r}$$

where $|r| < 1$

Evaluate

$$\sum_{n=1}^{7} 2^{n-1}$$

Evaluate

$5 + 0.5 + 0.05 + 0.005 + \cdots$

$$\sum_{n=0}^{\infty} 5\left(\frac{1}{2}\right)^n$$

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10-05 Mathematical Induction

- Proofs for _____formulas
- Show it works when _____
- Show it works for _____

Steps for Proof by Induction

1. Show it works for _____
2. _____formula works for $n = k$
3. Show it works for _____

- If proving _____formula use $S_{k+1} = S_k + a_{k+1}$

Prove $5 + 7 + 9 + 11 + 13 + \cdots + (3 + 2n) = n(n + 4)$

Prove $1(1 - 1) + 2(2 - 1) + 3(3 - 1) + \cdots + n(n - 1) = \frac{n(n-1)(n+1)}{3}$

Prove $(n+1)! > 2^n$ where $n \geq 2$

Prove 4 is a factor of $5^n - 1$

Precalculus

10-06 Binomial Theorem

$(x+y)^0$ 1

$(x+y)^1$ $1x$ $1y$

$(x+y)^2$ $1x^2$ $2xy$ $1y^2$

$(x+y)^3$ $1x^3$ $3x^2y$ $3xy^2$ $1y^3$

$(x+y)^4$ $1x^4$ $4x^3y$ $6x^2y^2$ $4xy^3$ $1y^4$

Properties

1. _____terms
2. Powers of *x* count _____, *y*'s count _____
3. Sum of exponents of each term = _____
4. Coefficients are _____

Binomial theorem

$$(a+b)^n = \sum_{r=0}^{n} {}_nC_r a^{n-r} b^r$$

- where ${}_nC_r = \frac{n!}{(n-r)!r!}$

Evaluate

${}_9C_2$ $\binom{11}{4}$

$\binom{8}{8}$ $\binom{4}{2}$

Expand $(x+2)^4$

Expand $(3 - x^2)^5$

Find the coefficient of the term a^4b^7 in $(2a - 3b)^{11}$

Precalculus

10-07 Counting Principles

Fundamental Counting Principle

- If events E_1 and E_2 occur in m_1 and m_2 ways, the number of ways _____events can occur is _____.

A lock will open with the right choice of 3 numbers. How many different sets of 3 numbers can you choose if each number is from 1 to 30 inclusive? (a) with repetition (b) without repetition

How many license plates can be made if each is 2 letters follow by 4-digits? (a) with repetition (b) without repetition

Permutation

- Number of ways to _____n objects taken r at a time

$$ {}_nP_r = \frac{n!}{(n-r)!} $$

How many ways can 8 children line up in a row?

A club has 24 members, how many ways can 5 officers be selected?

Distinguishable Permutations

- We want the orders that look _____(choosing _____the objects)

$$ \frac{n!}{q_1! \cdot q_2! \cdot q_3! \cdots} $$

- Where n = number of objects; q = how many times each is repeated

How many distinguishable ways to order the letters in BANANA?

Combinations

- Grouping of objects _____order

$$_nC_r = \frac{n!}{(n-r)!\,r!}$$

There are 31 students. How many different groups of 4 can be made?

You are forming a 10-person committee from 9 women and 12 men. How many different committees if 5 women and 5 men?

Precalculus

10-08 Probability

Probability

- Number from _____to _____indicating how _____something is to happen.
- 0 = _____happens
- 1 = _____happens

$$P(A) = \frac{favorable\ outcomes}{total\ outcomes}$$

A box contains 3 red marbles, 5 black marbles, and 2 yellow marbles. If a marble is selected at random, what is the probability of choosing yellow?

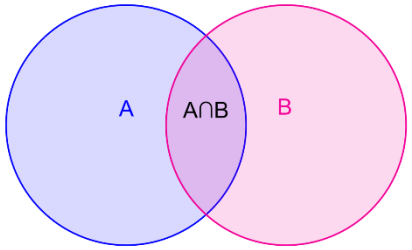
2 dice are rolled, what is the probability that the sum is 8?

Compound Events

- _____event with _____accepted outcomes

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

- If $P(A \cap B) = 0$, then called _____exclusive



You draw one card from a standard 52-card deck. What is the probability of drawing a heart or red?

Multiple Events

- _____events with _____outcomes
- Independent – Event A _____affect event B

$$P(A\ and\ B) = P(A) \cdot P(B)$$

- Dependent – Event A _____affect event B

$$P(A\ and\ B) = P(A) \cdot P(B|A)$$

  - where $P(B|A)$ is the probability that B occurs given that A already occured

You draw 2 cards from a standard 52-card deck. What is the probability you draw a heart and a red? (a) with replacement (b) without replacement

Complement

- _____

$$P(\overline{A}) = 1 - P(A)$$

- _____P(n ≥ 1) is easier with the complement _____$P(0)$